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Generation of Production Rules with Belief Functions to Train Fuzzy Neural Network in Diagnostic System

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Abstract—The article examines some algorithms for joint processing of raw data on the state of a complex multistage continuous production process to obtain probabilistic characteristics of abnormal critical events that can potentially lead to single failures or even emergencies. The article, thus, proposes and substantiates an approach to developing a technology to detect and predict malfunctions and determine their causes. The sequence of operations to process and convert diagnostic process data is considered essential. As a result, the article presents a general diagnostic model of a multistage production process. The model can formalize the main objects and processes in terms of the problem being solved. An incident is defined as an abnormal critical event described by non-normative values of diagnostic variables. Incidents are shown to be indicated by the corresponding membership functions. The hypotheses on potential incident causes are discussed to be built with belief functions being the basis of evidence theory or Dempster–Shafer theory. The hypotheses are characterized by an interval of malfunction probability in some process chain. The authors propose a procedure of converting these hypotheses into fuzzy production rules automatically. The automatical procedure is a prerequisite to using fuzzy neural networks to obtain a reliable estimate of the degree of belief in the incident cause. As a summary, the generated database of the production rules to train a neural network is substantiated to be used with the TSK architecture that makes possible to estimate a malfunction probability in the process chain quickly without resource-intensive computations.

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1. INTRODUCTION

Uncertainty in obtaining and analyzing data creates a problem of trust in the data used for decision-making, which may be critical for continuous production. One of the approaches to reduce uncertainty in the data used and to increase their degree of belief may be to hybridize the methods of processing fuzzy information intelligently. The difficult formal problem of reducing uncertainty is a significant problem induced by the adequacy of the methods used. The complexity of the required computations should also be taken into account.

The article examines the algorithms for joint processing of raw data on the state of a complex multistage continuous production process in order to obtain probabilistic characteristics of abnormal critical events that can potentially lead to single failures or even emergencies. As a summary, the authors

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propose and substantiate an approach to developing a technology to detect and predict malfunctions and to determine their causes.

The problem of determining the source of potentially possible abnormal situations can be verbalized as follows: to evaluate the probability of the fact that the cause of a certain abnormal situation is a malfunction of a certain unit of process equipment.

Further, the article systematically describes the approach proposed to solve the problem. A synopsis of publications is given and the place of the discussed approach in the related research is found. The sequence of operations to process and convert diagnostic process data is considered essential. As a result, a general diagnostic model of a multistage production process is presented. The model can formalize the main objects and processes in terms of the problem being solved. An incident is defined as an abnormal critical event described by non-characteristic values of diagnostic variables. Incidents are shown to be indicated with the corresponding membership functions. Further, the article discusses the hypotheses about potential incident causes. The hypotheses are built with belief functions being the basis of evidence theory or Dempster—Shafer theory. The hypotheses are formalized by an interval of malfunction probability at some process stage carried out by the corresponding production process chain. A procedure to convert the hypotheses into fuzzy production rules automatically is proposed. This is a prerequisite for using fuzzy neural networks to obtain a reliable estimate of the degree of belief in the incident cause. As a summary, the authors substantiate the use of the generated database of production rules to train a neural network with TSK architecture. This neural network will help to estimate a malfunction probability in the process chain quickly without resource-intensive computations. In conclusion, some features of the proposed approach are discussed.

2. RELATED WORKS

The review [1] describes in detail the main trends in the joint application of neural networks (NN) and evidence theory (ET) algorithms [2, 3]. The analysis makes it possible to identify the following main areas of research on the subject under consideration:

1. Using ET to prepare the data for creating and configuring NN. The data obtained from several sources are consolidated with the ET algorithms of combining evidences. Then, these combined data are fed to the NN input to extract the characteristics required to solve a specific problem. The approach is good for both training NN and using them in the operating mode. The approach is used, for example, in diagnostic systems [4] and autonomous driving systems [5, 6].

2. Using NN to collect raw data for ET. NN is used to generate the data on base probability distributions (or initial belief measures). The distribution of subjective probabilities is a key for executing the main ET algorithms. The result of NN work serves as an alternative to peer reviews. The approach is used, as a rule, to develop various classifiers [7, 8], as well as in predictive systems [9].

3. Combining NN and other machine learning methods with ET methods. A specific problem (e.g. device troubleshooting or object classification) is solved by different methods or by different variations of one method (for example, by different types of NN or NN and some fuzzy inference algorithm). Assuming that each of the methods used has its own limitations, we perform a complementary merge of their results. At the same time, the improved accuracy and reliability of the decisions made are expected to obtain. The approach is rather universal and is used in various decision-making systems: monitoring [10], medical diagnostics [11], equipment diagnostics [12], etc.

More detailed descriptions of the projects and research can be found in [1].

The approach presented in the article is the authors contribution to conducting research in the first direction of the three ones presented above. Other publications on certain aspects of the approach under discussion are noted further in the relevant sections of the article.

3. GENERAL MODEL OF PROCESS DIAGNOSTICS

Consider a multistage continuous *production process* $F = \{f_1, ..., f_n, ..., f_N\}$ with N process stages $f_n = (c_n, X_n, R_n)$, where n is a number of a process stage f_n; $c_n = \{c_{n1}, ..., c_{nu}, ..., c_{nU}\}$ is *a process chain (PC)* of production realizing a process stage f_n, $|c_n| = U$; c_{nu} is a single piece of process equipment with number u, used at a process stage f_n; $X_n = \{\overline{x}_{nm}\}$ is a set of *diagnostic variables* (DV) $\overline{x}_{nm}$, m is a sequence number for $\overline{x}_{nm}$, $m = 1, 2, ..., |X_n|$.

Note 1. All DV $\overline{x}_{nm}$ are output process variables which values are equipment sensor data. If we accept that one piece of PC equipment c_n is characterized by one base DV $\overline{x}_{nmu}$, values $\overline{x}_{nmu}$ determine the performance efficiency of an equipment piece with number u at the stage f_n.

$\overline{x}_{nm} = (o_{nm}, d_{nm}, \widetilde{x}_{nm})$ is a tuple where o_{nm} is unique name of DV, d_{nm} is an interval of DV normative values, $\widetilde{x}_{nm}$ is a DV value tuple; $\widetilde{x}_{nm} = (x_{nm1}, ..., x_{nmv}, ..., x_{nmV})$, $|\widetilde{x}_{nm}| = V$;

$x_{nm} = [\underline{x}_{nm}, \bar{x}_{nm}]$ are DV interval values of o_{nm} determining some actual state of F with o_{nm} at the output of c_n at the stage f_n for some period of time or observation interval $T = [\underline{t}, \overline{t}]$. Evidently, $\underline{x}_{nm} \leq \bar{x}_{nm}$.There are two ways of DV analysis. According to the first one when $\underline{x}_{nm} = \bar{x}_{nm}$, x_{nm} does not have an interval value but a real one. In this case, uncertainty is considered as a measurement error. The second method uses DV interval values and can be implemented with the interval analysis [13, 14].

$x_D = [\underline{x}, \overline{x}]$ is a domain of $\underline{x}_{nm}$ and $\bar{x}_{nm}$, where $\underline{x}$ and $\overline{x}$ are lower and upper limits of x_{nm} respectively. It is clear that $\underline{x} \leq \underline{x}_{nm} \leq \overline{x}_{nm} \leq \overline{x}$.

$d_{nm} = \left(\underline{d}_{nm}, \overline{d}_{nm}\right)$ is a tuple of DV normative (legitimate) values of o_m, defining the state of the normal performability of F with o_m at the output of c_n at the stage f_n; $\underline{d}_{nm}, \overline{d}_{nm}$ are lower and upper normative values of o_m respectively.

$R_n = \{r_{n1}, ..., r_{np}, ..., r_{nP})$ is a set of variables to control c_n, $|R_n| = P$.

Let the *incident on* o_m be some abnormal critical event defined by non-normative values x'_{nm} for o_m

$$X' = (x'_{nm}, \mu_I(x_{nm}))|x'_{nm} \in X' \subseteq X, \tag{1}$$

where $\mu_I(x_{nm}) \in [0, 1]$ is a membership function x'_{nm} for the set X'; $X = \bigcup_N x_n = \{x_{nm} : \exists n \; x_{nm} \in x_n\}$ is a set of all values of all DV (a base set).

Note 2. Generally, an incident may be defined by a simultaneous capture of x'_{nm} from different $\overline{x}_{nm}, m = 1, 2, ...$ having the same cause of x'_{nm}.

For each x'_{nm} some DV fuzzy set A where an incident may occur can be found. Thus

$$\forall P\left(x'_{nm}\right) \in P: \; P\left(x'_{nm}\right) = (0, 1] \rightarrow A\{(c_n; \mu_{nm}(c_n))\}, \quad A \neq \emptyset, \tag{2}$$

where $P = \{P(x'_{nm})\}$ is a set of indications that an incident has occurred; $C = \{c_n\}$ is a set of diagnosed c_n; $C \supseteq A$; $\mu_{nm}(c_n) = [0, 1]$ is a membership function of c_n for the set A or the degree of belief in $c_n \in A$.

Possible methods of computing membership functions $\mu_I(x_{nm})$ and $\mu_{nm}(c_n)$ are sure to be of interest to us.

4. DETECTING INCIDENTS

To detect an incident will mean to indicate abnormal critical events by computing a membership function $\mu_I(x_{nm})$. For real values x_{nm} we will introduce a membership function $\mu_I^1(x_{nm})$ as follows

$$\mu_I^1(x_{nm}) = \begin{cases} 1 \quad \text{when } x_{nm} \leq \underline{d}_{nm}, \\ 1 - \frac{x_{nm} - \underline{d}_{nm}}{a} \quad \text{when } x_{nm} > \underline{d}_{nm} \wedge x_{nm} < (\underline{d}_{nm} + a), \\ 0 \quad \text{when } x_{nm} \geq (\underline{d}_{nm} + a) \wedge x_{nm} \leq (\overline{d}_{nm} - a), \\ 1 - \frac{\overline{d}_{nm} - x_n m}{a} \quad \text{when } x_{nm} > (\overline{d}_{nm} - a) \wedge x_{nm} < \overline{d}_{nm}, \\ 1 \quad \text{when } x_{nm} \geq \overline{d}_{nm}. \end{cases} \tag{3}$$

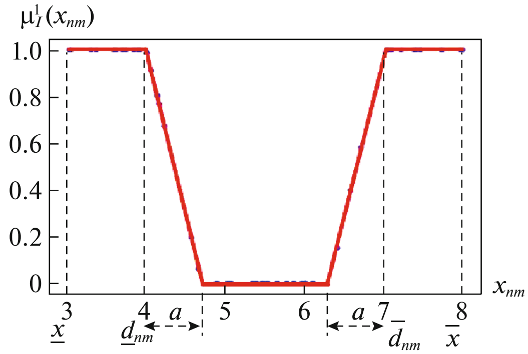

Fig. 1. Membership function $\mu_I^1(x_{nm})$.

Here, a allows for a measurement error and define the domain of partial membership of x_{nm} for the fuzzy set X'. Suppose that value a depends on the spread in values x_{nm} (being quite reasonable). Then,

$$a = k\sigma_{nm}^2 = k\frac{1}{V}\sum_{v=1}^{V}(x_{nmv} - \widehat{x}_{nmv})^2, \quad (4)$$

where $k = [0, 1]$ is a coefficient assigned in terms of actual values of x_{nm}; σ_{nm}^2 is a dispersion of x_{nm} computed for the set X of measured values x_{nm}; V is a number of measured values x_{nm}; x_{nmv} is the next vth measured value of x_{nm}; $\widehat{x}_{nmv} = \frac{1}{V}\sum_{v=1}^{V} x_{nmv}$.

It is evident that the function $\mu_I^1(x_{nm})$ is a common piecewise linear trapezoidal membership function of the first order.

Example 1. Figure 1 shows the graphical interpretation of the function $\mu_I^1(x_{nm})$. The following input data are used: $|X_n| = 1$, $\underline{x} = 3.0$, $\overline{x} = 8.0$, $\underline{d}_{nm} = 4$, $\overline{d}_{nm} = 7$, $k = 0.3$.

For the interval values $x_{nm} = [\underline{x}_{nm}, \bar{x}_{nm}]$ the membership function $\mu_I^2(x_{nm})$ is introduced as follows

$$\mu_I^1(x_{nm}) = \begin{cases} 1 \quad \text{when } \bar{x}_{nm} \le \underline{d}_{nm}, \\ \frac{\underline{d}_{nm} - \underline{x}_{nm}}{\bar{x}_{nm} - \underline{x}_{nm}} \quad \text{when } \underline{x}_{nm} < \underline{d}_{nm} \wedge \bar{x}_{nm} > \underline{d}_{nm}, \\ 0 \quad \text{when } \underline{x}_{nm} \ge \underline{d}_{nm} \wedge \bar{x}_{nm} \le \overline{d}_{nm} \\ \frac{\bar{x}_{nm} - \overline{d}_{nm}}{\bar{x}_{nm} - \underline{x}_{nm}} \quad \text{when } \underline{x}_{nm} < \overline{d}_{nm} \wedge \bar{x}_{nm} > \overline{d}_{nm}, \\ 1 \quad \text{when } \underline{x}_{nm} < \underline{d}_{nm} \wedge \bar{x}_{nm} > \overline{d}_{nm}, \\ 1 \quad \text{when } \underline{x}_{nm} \ge \overline{d}_{nm}. \end{cases} \quad (5)$$

Example 2. Figure 2 shows the graphical interpretation of the function $\mu_I^1(x_{nm})$. The following input data are used: $V = 1000$, $|X_n| = 1$, $\underline{x} = 3.0$, $\overline{x} = 8.0$, $\underline{d}_{nm} = 4$, $\overline{d}_{nm} = 7$. The interval values $x_{nm} = [\underline{x}_{nm}, \bar{x}_{nm}]$, i.e. the members of the set $\widetilde{x}_{nm} = \{x_{nm}\}$, are generated as pseudorandom numbers from the interval $x_D = [\underline{x}, \overline{x}]$.

The interval $[\underline{d}_{nm}, \underline{d}_{nm} + \max(\bar{x}_{nm} - \underline{x}_{nm})] \mid \underline{x}_{nm} < \underline{d}_{nm} \wedge \bar{x}_{nm} > \underline{d}_{nm}$ and the interval $[\max(\bar{x}_{nm} - \underline{x}_{nm}), \overline{d}_{nm}] \mid \underline{x}_{nm} < \overline{d}_{nm} \wedge \bar{x}_{nm} > \overline{d}_{nm}$ define the domains of uncertainty for $\mu_I^1(x_{nm})$.

Basic properties of an incident fuzzy set $X' = \{(x'_{nm}, \mu_I(x_{nm}) | x'_{nm} \in X'\}$: $X_S{}' = \{x_{nm} | \mu_I(x_{nm}) > 0, x_{nm} \in X\}$ is a support of X'. $X_S{}'$ involves the measurements of non-normative values of diagnostic variable x_{nm}. $X_C{}' = \{x_{nm} | \mu_I(x_{nm}) = 0, x_{nm} \in X\}$ is a core of X'. $X_C{}'$ involves the measurements of normative values of diagnostic variable x_{nm}.

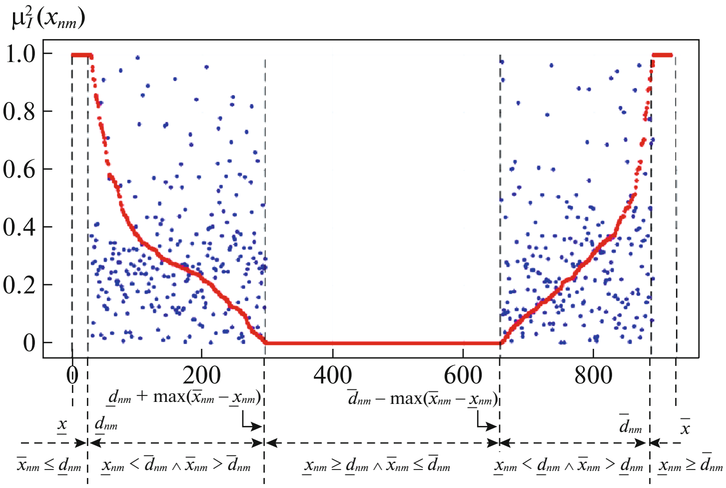

Fig. 2. Membership function $\mu_I^2(x_{nm})$.

5. GENERATING HYPOTHESES ON POTENTIAL INCIDENT CAUSES

The presence of a non-empty incident set X' results from the normal operation malfunction in a single or several $c_n \in A$. Meanwhile, $\mu_{nm}(c_n)$ is a membership function of c_n for the set A or the degree of belief in $c_n \in A$. To define $\mu_{nm}(c_n)$ we propose to use the ET algorithms [15]. The approach will be briefly examined here, more details can be found in [16, 17].

Any subsets $x' \subseteq X'$ including individual incidents x'_{nm} being considered as *events* may have some measures of belief in them or *base probabilities* of incidents. Let $C = \{c_n\}$ be some finite set of diagnosed PC; $C = \{c_n\}$ be any subset of potentially malfunctioning PC; $C = \{c_n\}$ be a power set, $|S(C)| = n_A$. A base probability is defined as follows

$$m: \ S(C) \to [0, 1],$$
$$\sum\nolimits_{i=1|}^{n_A} m(A_i) = 1, \quad A_i \in S(C). \tag{6}$$

A set of values $m(A_i)$ form the distribution of base probabilities $m(A)$. If $m(A_i) > 0$, then A_i is referred to as a focal element, and a pair $(A_i, m(A_i))$ is referred to as a body of evidence. If $m(\emptyset) = 0$, then the distribution of $m(A)$ will be normalized. The value $m(\emptyset) > 0$ for non-normalized distributions is interpreted as the degree of belief in $c_n \notin C$, i.e. the malfunction cause is not in PC being diagnosed.

The degree of belief in the incident cause being $c_n \in A_i$ is defined with a *belief function*

$$Bel(A_i) = \sum_{A_j \subseteq A_i} m(A_j), \tag{7}$$

where $A_i \in S(C)$ is a focal element; $m(A_j) \in [0, 1]$ is the element mass of $A_j \subseteq A_i$ indicating the possible incident cause in $c_n \in A_j$ but not generating any additional statements about other focal elements. $Bel(A_i)$ has the following properties: $Bel(\emptyset) = 0$, $Bel(A_i) \in [0, 1]$ and $Bel(S(C)) = 1$. The value $Pl(A_i) = 1 - \sum_{A_i \cap A_j = \emptyset} m(A_j)$ is referred to as *a plausibility function*.

According to (6) $Bel(A_i)$ and $Pl(A_i)$ can be seen as lower and upper bounds of $\mu_{nm}(c_n)$ — the membership function c_n for the set A_i or the degree of belief in the incident cause being $C_n \in A_i \subseteq S(C)$

$$Bel(A_i) \leq \mu_{nm}(c_n) \leq Pl(A_i) \quad \text{or} \quad \mu_{nm}(c_n) = [Bel(A_i), Pl(A_i)]. \tag{8}$$

It should be noted that the value of membership function $\mu_{nm}(c_n)$ in the present case is an interval number. Let us call the interval of function values $\mu_{nm}(c_n) = [Bel(A_i), Pl(A_i)]$ the *incident cause hypothesis*. Thus, many incident cause hypotheses for all $A_i \subseteq S(C)$ can be built.

An important problem is to obtain initial values of base incident probabilities of $m(A_i)$ for each A_i. The detailed discussion is beyond the scope of this article. We should only note that assumptions about

Table 1. Incident cause hypotheses for pairs: values of belief functions $Bel(A_i)$ and plausibility functions $Pl(A_i)$

		Incident on o_{11}		Incident on o_{12}		Incident on (o_{11}, o_{12})	
PC	PC name	$Bel(A_i)$	$Pl(A_i)$	$Bel(A_i)$	$Pl(A_i)$	$Bel(A_i)$	$Pl(A_i)$
c_1	Gas communication of air supply on technology and auxiliaries of gas technological turbocharger (GTT)	0.312	0.813	0.094	0.339	0.222	0.4
c_2	Gas communication of ammonia supply in a gas blender	0.125	0.626	0.283	0.528	0.302	0.48
c_4	Gas communication of nitrous gases (NG) in an absorption tower	0.0	0.563	–	–	0.0	0.2
c_6	Gas communication of air supply to absorption	0.0	0.563	–	–	0.0	0.2
c_8	Hydraulic communication of weak nitric acid (WNA)	–	–	0.378	0.623	0.275	0.453

the causes of abnormal values of x'_{nm}, associated with malfunction of PC c_n, can be obtained from the following sources: peer reviews; statistical data; production procedures.

Based on the peer reviews, the incident for a given DV is evaluated by an expert as $m_e(A_i) = [0, 1]$, with the expert assessing the set A_i.

Based on incident statistics, the data should be represented by the incident experimental distribution (a histogram) for a given DV for A_i. Base probabilities of $m(A_i)$ are generated with known algorithms. For example, the use of relative incident frequencies with additive smoothing gives

$$m(A_i) = \frac{n_i + \lambda}{n + \lambda n_A}, \tag{9}$$

where n is the number of recorded incidents; n_i is the number of incidents which cause is the malfunction of PC from A_i; $\lambda > 0$ is smoothing (usually $\lambda = 0$).

Some authors also propose to use imprecise Dirichlet model [18], belief maximization by solving the linear programming problem including special cases of analytical solution [19, 20].

Based on the analysis of production procedures, the possibility to clarify information about PC probable malfunctions as the incident causes opens up.

It should be noted that the need to combine the bodies of evidence on the incident obtained from different sources is obvious. There are Dempster's denormalized and normalized conjunctive rules of combination, a disjunctive rule of combination, etc. The details on the combination rules can be found in [21]. The key to combining the bodies of evidence is their internal and external conflict arising frequently from the objective information inconsistency. The analysis of the evidence conflict, the methods to construct and formalize conflict measures are detailed in review [22].

Example 3. Table 1 shows the example of a final combination of hypotheses on incident causes. The combination is the result of simulating a multistage chemical-technological process with a special demonstrator [23]. Here are some explanations. The raw data of the incident on $o_{11} = 'X5'$: $x'_{11} = [\underline{x}_{11}, \bar{x}_{11}] = [0.35, 0.65]$, $d_{11} = \left(\underline{d}_{11}, \overline{d}_{11}\right) = (0.5, 0.8)$, $\mu^1_I(x_{11}) = 0.5$; the incident on $o_{12} = 'Z1'$: $x'_{12} = [\underline{x}_{12}, \bar{x}_{12}] = [46.0, 51.3]$, $d_{12} = \left(\underline{d}_{12}, \overline{d}_{12}\right) = (50.0, 55.0)$, $\mu^1_I(x_{11}) = 0.755$; $A = \{c_1, c_2, c_4, c_6, c_8\}$. Having done all the transformations in accordance with the TE methods including the computation of the normalized conjunctive combination of the bodies of evidence for the given DV of o_{11} and o_{12}, we obtain focal elements $A_1 = \{c_1\}$, $A_2 = \{c_2\}$, $A_3 = \{c_4, c_6\}$, $A_4 = \{c_8\}$, $A_5 = \{c_1, c_2, c_4, c_6, c_8\}$; the bodies of evidence $(A_1, m(A_1) = 0.222)$, $(A_2, m(A_2) = 0.302)$, $(A_3, m(A_3) = 0.022)$, $(A_4, m(A_4) = 0.275)$, $(A_5, m(A_5) = 0.178)$; $K = 0.375$.

6. GENERATING PRODUCTION RULES

The incident cause hypotheses built on the given DV can be described with the production rules (hereinafter referred to as rules) having the following general view

$$R_i: \ IF \ \exists x'_{nm} \in X' \ THEN, \quad \exists A \neq \emptyset, \tag{10}$$

where R is the rule, $i = 1, 2, ...G$; G is the total number of rules; X' is, according to (1), a fuzzy set of incidents with a membership function $\mu_I(x_{nm}) \in [0, 1]$; A is, according to (2), a fuzzy set of potential incident PC with a membership function $\mu_{nm}(c_n) = [Bel(A), Pl(A)]$.

Rule (10) verbally goes like this: *if an incident has occurred at any production stage, then its cause is some fuzzy set of PC*. The combination of such rules is a fuzzy logic model — a knowledge base usually developed by domain experts. Our purpose is to show how such a knowledge base can be generated automatically with the hypotheses on incident causes built with belief functions.

More classically, fuzzy rules (10) are written as

$$R_i: \ IF \ x \ is \ \widetilde{A}_i \ THEN \ y \ is \ \widetilde{B}_i, \tag{11}$$

where x and y are input and output variables respectively; $\widetilde{A}_i$ and $\widetilde{B}_i$ are fuzzy sets with the corresponding membership functions.

Let us make the following transformation of the structure of the rules (11), with **Note 2** taken into account

$$R_i\text{: } IF \ \mu_I(o_{n1}, x'_{n1}) \ AND \ \mu_I(o_{n2}, x'_{n2}) \ AND ... AND \ \mu_I(o_{nm}, x'_{nm}) \ THEN \ \mu_{nm}(c_n), \tag{12}$$

where $\mu_I(o_{nm}, x'_{nm})$ means the degree of belief in the non-characteristic value of x'_{nm} having led to the incident on DV o_{nm}; $(o_{n1}, ..., o_{nm}) = o'_n$ is a tuple of input linguistic variables — DV names; $(x'_{n1}, ..., x'_{nm}) = x'$ is a tuple of non-characteristic values of input DV, $x'_n \subseteq X'$, X' is an antecedent domain of rule R_i; $c_n \subseteq A$ is a linguistic variable determining malfunctional PC (the incident cause). A is a consequent domain of rule R_i (all potentially malfunctional PC); $\mu_{nm}(c_n) = [Bel(A), Pl(A)]$. Then, one gets from (11) and (12)

$$R_i^{cn}: \ IF \ x_1 = \mu_{n1} \ AND \ x_2 = \mu_{n2} \ AND ... AND \ x_m = \mu_{nm} \ THEN \ y = \mu_{cni}, \tag{13}$$

where R_i^{cn} is the rule for c_n, $i > 0$ is the rule number, c_n is PC at the process stage f_n; x_m is the degree of belief in non-characteristic value of mth DV being an incident indicator (input variable); μ_{nm} is the value of x_m; y is the degree of belief that the incident cause is a malfunction in c_{ni} (output variable); μ_{cni} is the value of y, $\mu_{cni} = (Bel(A), Pl(A))/2$ (the interval center $[Bel(A), Pl(A)]$). The value is sure to be within $[Bel(A), Pl(A)]$ and is a reliable estimate of the hypothesis c_n.

Thus, the conclusion is obvious: having the values of the belief and plausibility functions computed, we can automatically generate a database of production rules for all the hypotheses on the incident causes.

Example 4. The example of a production rule database developed with the computation of belief and plausibility functions (with the data from Example 3 for PC c_1)

$$R_1^{c1}: \ IF \ x_1 = 0.755 \ THEN \ y = 0.217,$$
$$R_2^{c1}: \ IF \ x_2 = 0.5 \ THEN \ y = 0.563,$$
$$R_3^{c1}: \ IF \ x_1 = 0.755 \ AND \ x_2 = 0.5 \ THEN \ y = 0.311. \tag{14}$$

7. USING TSK NEURAL NETWORK

We propose to use a database of production rules (13) for training a fuzzy neural network with ANFIS (adaptive network-based fuzzy inference system) based on Takagi–Sugeno–Kang model (TSK) [24].

The TSK network implements a fuzzy production model based on the following rules

$$\begin{aligned} R_i: \ & IF \ x_1 \ is \ \widetilde{A}_{i1} \ AND \ x_2 \ is \ \widetilde{A}_{i2} \ AND \ x_m \ is \ \widetilde{A}_{iM} \\ & THEN \ y_i = c_{i0} + c_{i1}x_1 + c_{i2}x_2 + ... + c_{iM}x_M, \end{aligned} \tag{15}$$

where $i = 1, 2, ...G$ is a rule number; $x_1, x_2, ..., x_m, ..., x_M$ are input real variables, $m = 0, 1, ...M$; $\widetilde{A}_{i1}, \widetilde{A}_{i2}, ..., \widetilde{A}_{im}, ..., \widetilde{A}_{iM}$ are fuzzy sets; y_i is an output real variable; $c_{i0}, c_{i1}, ..., c_{im}, ..., c_{iM}$ are the consequent parameters for R_i (crisp numbers).

It is not difficult to see that rules (13) are similar to rules (15) by their structure. In (15) x_m is interpreted as the degree of belief in the fact that a non-normative value of mth DV has led to some incident and y_i — is the degree of believe that the incident cause is PC c_n malfunction for which rule R_i is defined. As shown above in Sections 4 and 5, values $x_m \sim \mu_{nm}$ and $y_i \sim \mu_{cni}$ define the membership of some elements to fuzzy sets. Therefore, the initial problem of defining a reliable estimate of PC malfunction probability as a diagnostic cause can be solved with TSK fuzzy neural network. Moreover, the network can be trained with a database of production rules developed automatically for each incident cause hypothesis built with functions of belief. TSK network has five layers

- Layer 1. Fuzzification of R_i. The following membership functions $\mu(x_m)$: are usually used: a generalized bell one $\mu(x_m) = 1/1 + \left[((x_m - c)/a)^2\right]^b$ or Gaussian one $\mu(x_m) = \exp\left[-((x_m - c)/a)^2\right]$. In both functions a, b and c represent arbitrary real numbers.

- Layer 2. Aggregation of x_m for each rule R_i: $w_i = \prod_{m=1}^{M} \mu(x_m)$.

- Layer 3. Evaluation of functions $y_i(x) = p_{i0} + \sum_{m=1}^{M} p_{im} x_m$ for each rule R_i.

- Layer 4. Summing of $s_1 = \sum_{i=1}^{G} w_i \times y_i(x)$ and $s_2 = \sum_{i=1}^{G} w_i$.

- Layer 5. Defuzzification of the result: $y(x) = s_1/s_2$.

Layer 1 is used to configure a, b and c of function $\mu(x_m)$ (a and c in case of a Gaussian function), Layer 2 is used to configure p_{i0} and p_{im} of functions $y_i(x)$. The total number of parameters configured in training the network is $G(4M + 1)$. A training data set is generated automatically as shown in Section 6. A basic algorithm to train TSK network is given in [24]; other algorithms can also be used (e.g. [25] proposing to combine fuzzy inference systems with a genetic algorithm).

8. DISCUSSION

1. The model of the multistage production process diagnostics described in Section 2 is simplified to a certain extent due to the real structural and functional characteristics. In particular, the issues of the process stage decomposition and the hierarchy of a decision-making system are not taken into account. However, it should be noted that this kind of detail is not necessary for the problem set and its solution described in the article. The discussed methods of detecting incidents, forming hypotheses about their potential causes, generating production rules for training a neural network are quite accurately described by the parameters of the proposed model.

2. The membership function $\mu_I(x_{nm})$ for the fuzzy incident set X' used to detect incidents and described in Section 4 is represented in the two variants: for real and interval values of DV. The functions $\mu_I^1(x_{nm})$ and $\mu_I^2(x_{nm})$ are chosen upon the authors previous papers ([16, 17], etc.), the choice being supported empirically. The problem specification and the *incident* concept may have proposed a more reasonable version of the function $\mu_I(x_{nm})$ such as the modification of a generalized bell membership function: $\mu(x_{nm}) = 1/1 + \left[((x_{nm} - c)/a)^2\right]^b$. The continuity of x_{nm} value transitions from non-membership to membership in set X', guaranteed in this case, makes it possible to simulate the spread of DV measurements successfully. In terms of interval values $x_n m$, the membership function $\mu(x_{nm})$ will form a fuzzy set of the second order and consider DV interval fuzziness. Otherwise, it is difficult to determine reasonably the exact value of the function $\mu(x_{nm})$ for a fuzzy incident set X'. Additional research is required to consider these issues.

3. Section 5 describes that one of the sources of incident cause assumptions and computations of base probabilities $m(A_i)$ is based on statistical data. In terms of DV values obtained from sensors, the

consistency of these data with the data on equipment malfunctions is mandatory. At the same time, the sampling representativeness should be ensured, which is possible only with a long service life.

4. Consider one of the arguments discussed in Section 6. Let G^{cn} is the number of production rules to train the neural network of the given PC c_n. Then, G^{cn} is equal to the number of incidents which causes are include by the hypotheses in the malfunction probability in c_n. In other words, $(\exists \mu_{nm}(c_n) : x'_{nm} \to \{A_i\} \, |m = 1, 2, ..., |X_n|, x'_{nm} \in X'_{nm}, c_n \in A_i)G^{cn} = |X'_{nm}|$. The number of must not be large; the incident, as a rule, is diagnosed by finite and small number of DV.

5. The implementation of the proposed approach is assumed to be the development of the project [23]. The sequence of procedures to prepare a training data set for a neural network is as follows: a) to generate combinations of various DV values from the actual sensor data or the values generated from specified spread; b) to generate a fuzzy incident set in accordance with (1); c) to build the hypotheses on potential incident causes as determined by (8); d) to generate the production rules having a structure (13); e) to prepare a data array with the actual antecedent and consequent values of the rules (13). When the neural network accuracy is evaluated, a fuzzy incident set should be generated with an algorithm different from the one used to obtain the training set (if the sensor data were not used).

6. There is no complete clarity on the issues of retraining the neural network under discussion. The proposed approach should be verified experimentally on the real data of process and equipment diagnostics. At the same time, the retraining algorithm should be chosen.

9. CONCLUSIONS

We can draw a general conclusion that the problem as a whole has been solved. The formalization of the main objects and multistage production process in the general model of its diagnostics made it possible to define the concept of an incident and to show how incidents are indicated. As a result, a fuzzy set of incidents is generated with the corresponding membership functions correlated with their base probabilities. The hypotheses on potential incident causes in terms of base incident probabilities and belief functions are automatically converted into a database of production rules to train a fuzzy neural network in order to obtain a reliable estimate of the degree of belief in the incident cause.

The proposed approach allows for quick computations of an estimated probability of malfunctions in process chains without expensive computing resources. The effectiveness of the proposed approach certainly requires experimental verification, but the combination of theoretically sound methods discussed in the article gives hope for a positive perspective.

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