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ARTICLE

On the classification of smooth Fano weighted complete intersections

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Abstract
We provide a complete classification of smooth Fano n-folds, $6 \leq n \leq 10$, such that their images under their (sub)-anticanonical embedding can be described as a wellformed weighted complete intersection of some weighted projective space and they are not intersections with a linear cone. In total, we obtain 1180 families of such Fano manifolds by utilizing a combination of algorithmic and theoretical methods.

Keywords Fano varieties · Weighted projective space · Complete intersection

Mathematics Subject Classification 14J45 · 14M10 · 14M07 · 14Q15

1 Introduction

1.1 Fano weighted complete intersections

A projective variety X with ample anticanonical divisor $-K_X$ is called a *Fano variety*. The *Fano index* i_X of X is the largest positive integer such that $-K_X = i_X D$ for some ample Cartier divisor D. Recall that a closed subvariety

$$X = \mathbb{V}(f_1, \ldots, f_c) \subset \mathbb{P}(a_0, \ldots, a_N)$$

is a *weighted complete intersection (WCI) of multidegree* $(d_1, \ldots, d_c)$ if:

(i) Each $f_j \in \mathbb{C}[x_0, \ldots, x_N]$ is weighted homogeneous of degree $\deg(f_j) = d_j$ with $\deg(x_i) = a_i > 0$;
(ii) The weighted homogeneous polynomials $f_1, \ldots, f_c$ form a regular sequence in the polynomial ring $\mathbb{C}[x_0, \ldots, x_N]$, i.e. codimension$(X) = c$.

We write

$$X = X_{d_1,\ldots,d_c} \subset \mathbb{P}(a_0, \ldots, a_N)$$

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to denote a WCI in $\mathbb{P}(a_0, \ldots, a_n)$ and

$$X = \operatorname{Proj}\left(\frac{\mathbb{C}[x_0, \ldots, x_N]}{I}\right), \quad I = (f_1, \ldots, f_c).$$

A WCI X is *quasismooth* if its affine cone $\widetilde{X} \subset \mathbb{A}^{N+1} \setminus \{0\}$ is a smooth and it is *wellformed* if it does not contain a codimenion $c+1$ orbifold stratum of $\mathbb{P}(a_0, \ldots, a_N)$. If $d_j \neq a_i$ for all $1 \leq j \leq c$ and $0 \leq i \leq N$, then we say that it is *not an intersection with a linear cone* of $\mathbb{P}(a_0, \ldots, a_N)$. Any Fano variety in this paper is wellformed, smooth (in usual sense) and not an intersection with a linear cone of $\mathbb{P}(a_0, \ldots, a_N)$.

If

$$X_{d_1,\ldots,d_c} \subset \mathbb{P}(a_0, \ldots, w_n)$$

is a wellformed and quaismooth WCI X then by [14, 6.14] we know that

$$\mathcal{O}_X(K_X) = \mathcal{O}_X\left(\sum_{j=1}^{c} d_j - \sum_{i=0}^{N} a_i\right).$$

Moreover, if dimension of X is greater than or equal to 3 and it is a wellformed *Fano* variety then $\sum_j d_j < \sum_i a_i$ and the *Fano index* is $i_X = \sum_i a_i - \sum_j d_j$, by [26, Sec. 5].

When $n \geq 3$, the weighted Mori–Lefschetz theorem [22, Theorem 3.7] implies that a WCI X has $\operatorname{Pic}(X) \cong \mathbb{Z}$, generated by $\mathcal{O}_X(1)$.

1.2 Context and motivation

Fano varieties occupy a central place in birational and biregular geometry. In fixed dimension, it is known that the smooth Fano varieties form a bounded class [19] of varieties, providing a motivation to address the classification of smooth Fano varieties. In low dimensions, the classification is complete, with 1, 10, and 105 families of smooth Fano varieties in dimensions 1, 2, and 3 [15–17, 20, 21]. For dimension 4, only partial results are known; the index ≥ 2 case gives 35 families [10–12, 23, 24, 36, 37]. Batyrev found 123 toric Fano fourfold families [1], and some further examples arise as toric complete intersections [7, 8], quiver-flag zero loci [18], and Picard-rank-two Cox rings [13]. Earlier index-one cases due to Küchle came from homogeneous bundles and weighted complete intersections, while related Gorenstein-format families of small volume were constructed in [34].

For WCIs in weighted projective spaces, general constraints include a codimension bound for quasismooth WCIs [5] and degree bounds in terms of canonical invariants [6]. Afterward, Przyjalkowski–Shramov [26] established effective universal bounds on the weights of $\mathbb{P}(a_0, \ldots, a_N)$ and equation degrees d_j for smooth wellformed Fano WCIs. By using these bounds, they carry out a finite search to classify all smooth Fano WCIs in dimensions 4 and 5 and compute their basic invariants. Further properties of weighted complete intersections have been later explored in [28–30].

1.3 Main result

We use the bounds given by Przyjalkowski–Shramov [26] and the algorithmic approach developed in [3, 32] to provide a complete classification of smooth wellformed WCI of dimension $6 \leq n \leq 10$. In other words, we classify smooth Fano varieties (Fano manifolds) of dimension $6 \leq n \leq 10$ such that their images under their sub(anticanonical) embeddings

Table 1 Number of deformation families of smooth Fano weighted complete intersections for $6 \leq n \leq 10$

n	6	7	8	9	10
# families	72	108	201	292	507

can be described as weighted complete intersection in a weighted projective space and they are not intersections with a linear cone.

Theorem 1.1 *Let X be a smooth Fano variety of dimension $6 \leq n \leq 10$ that, under its (sub)anticanonical embedding, is a weighted complete intersection in weighted projective space and it is not an intersection with a linear cone. Then for each n, Table 1 lists the number of distinct deformations.*

The proof initiates with a candidate search for the possible smooth Fano WCIs of fixed codimension and Fano index, by using an algorithmic approach developed in [3, 32]. The algorithm uses the Hilbert series of a WCI to primarily identify if the Hilbert series may correspond to a smooth variety or otherwise. The search is finite due to bounds and further results provided in [25–27]. Then each candidate is checked if it indeed misses the orbifold locus of the ambient weighted projective space and base locus of the linear systems induced by hypersurfaces of degrees $d_1, \ldots, d_c$. We explain the details of various steps in Sect. 2.

2 Steps of proof

2.1 Bounds on computer search

Let

$$X_{\mathbf{d}} = X_{d_1,\ldots,d_c} \subset \mathbb{P}(a_0, \ldots, a_N) = \mathbb{P}(\mathbf{a}), \qquad N = n + c,$$

be a smooth wellformed Fano weighted complete intersection that is not an intersection with a linear cone. Then Przyjalkowski–Shramov provided the following bound on the weights of $\mathbb{P}(\mathbf{a})$.

Theorem 2.1 [26] *In the above setup,*

$$a_i \leq N \quad \text{for all } i = 0, \ldots, N, \qquad d_j \leq N(N+1) \quad \text{for all } j = 1, \ldots, c.$$

Theorem 2.2 [31] *In the above setup, at least $c + 1$ of the weights a_i are equal to* 1.

Since we perform our computer search in order of increasing sum of the weights $S = \sum a_i$ on the ambient weighted projective space, we need to have a bound to find all possible smooth Fano WCIs of given codimension and Fano index. By Theorem 2.2 at least $c+1$ weights are 1, and by Theorem 2.1 every other weight is at most $N = n + c$. Hence

$$S \; \leq \; (c+1) \cdot 1 \; + \; n \cdot (n+c).$$

Equivalently, for fixed dimension n and codimension c,

$$S \; \leq \; n^2 + (n+1)c + 1.$$

For each dimension $6 \leq n \leq 10$ and codimension c, we obtain the explicit upper bounds

$$\begin{aligned} n = 6 : S &\leq 6^2 + (6+1)c + 1 = 37 + 7c, \\ n = 7 : S &\leq 7^2 + (7+1)c + 1 = 50 + 8c, \\ n = 8 : S &\leq 8^2 + (8+1)c + 1 = 65 + 9c, \\ n = 9 : S &\leq 9^2 + (9+1)c + 1 = 82 + 10c, \\ n = 10 : S &\leq 10^2 + (10+1)c + 1 = 101 + 11c. \end{aligned} \tag{1}$$

2.2 Generating candidates

In this step, we generate the list of all possible Hilbert series that can give rise to a smooth Fano WCI. The algorithm primarily uses the theorem of Buckley–Reid–Zhou [4, Theorem 1.3] which, for an algebraic subvariety V of dimension greater than or equal to 2 of a weighted projective space with at worst isolated singularities, provides the parsing of the Hilbert series $P_V(t)$ into smooth part and the singular part. More specifically if V contains a basket of k types of isolated orbifold points

$$\mathcal{B} = \left\{ Q_i := \frac{1}{r_i}(a_i, b_i, c_i) | i = 1 \dots k \right\},$$

then

$$P_V(t) = P_{\text{smooth}}(t) + \sum_{i=1}^{k} m_i \times P_{Q_i}(t),$$

where m_i is the multiplicity of each singular point Q_i. In particular, V is smooth iff $P_V(t) = P_{\text{smooth}}(t)$ (equivalently, the basket is empty). The detailed nature of these terms in this decomposition can be found in [4].

We call a WCI Fano variety to be of type $\mathcal{X} := (n, c, i_x)$ if its dimension is n, codimension is c, Fano index is i_X, and it is not an intersection with a linear cone of $\mathbb{P}(\mathbf{a})$. We use the bounds (1) to search for all smooth Fano WCIs of type $\mathcal{X}$. We describe the algorithm below for the smooth Fano complete intersections in weighted projective spaces. In general, it also works for other types of varieties (non-Fano and singular) inside more general ambient varieties (weighted Grassmannains, etc.); see [2, 32, 33] for more details.

2.2.1 Algorithm

(i) *Ambient data* For $\mathbb{P}(\mathbf{a})$, such that the weights of $\mathbb{P}(\mathbf{a})$ satisfy the bounds (1), we record

$$P_{\mathbb{P}}(t) = \frac{1}{\prod_{j=0}^{N}(1 - t^{a_j})}, \qquad K_{\mathbb{P}} \cong \mathcal{O}_{\mathbb{P}}\left(-\sum_{j=0}^{N} a_j\right).$$

(ii) *Enumerate admissible WCIs of type $\mathcal{X}$*. Using the bounds (1), list all degree vectors $\mathbf{d} = (d_1, \dots, d_c)$ with

$$i_X = \sum_{j=0}^{N} a_j - \sum_{\ell=1}^{c} d_\ell, \qquad d_\ell \neq a_j \ \ \forall j, \ell,$$

so that $K_X \cong \mathcal{O}_X(-i_X)$ and X is not a linear-cone intersection.

(iii) *Hilbert series of* X **and its smooth part**. For each admissible $\mathbf{d}$, we compute the WCI Hilbert series

$$P_X(t) = \frac{\prod_{\ell=1}^{c}(1-t^{d_\ell})}{\prod_{j=0}^{N}(1-t^{a_j})} = \sum_{i\geq 0} c_m t^m.$$

From the first $\lfloor \frac{c}{2} \rfloor + 1$ coefficients of $P_X(t)$, one can compute the smooth part $P_{\text{smooth}}(t)$. If

$$P_X(t) - P_{\text{smooth}}(t) \equiv 0,$$

then we get a candidate smooth Fano WCI $X_{\mathbf{d}} \subset \mathbb{P}(\mathbf{a})$, otherwise we reject the Hilbert series.

(iv) *Output*. We run the step (iii) for each $\mathbf{d}$ computed at step (ii). The resulting list consists of all candidate smooth Fano WCIs $X_{\mathbf{d}} \subset \mathbb{P}(\mathbf{a})$ of type $\mathcal{X}$.

We get all Hilbert series that can arise from smooth Fano WCIs of type $\mathcal{X}$ by running all the steps of the algorithm for each set of weights $\mathbf{a}$ that satisfies the bounds (1).

2.3 Verifying smoothness

For each candidate WCI Fano variety X, we then check smoothness using the defining equations of X, which consists of two parts. We examine if the intersection of X with the orbifold locus of the ambient weighted projective space is empty. Also by Bertini's theorem the singularities may lie on the reduced part of the base loci of the linear systems $|\mathcal{O}(d_i)|$, $1 \leq i \leq c$. In case the base loci do not intersect X, we can conclude the variety to be smooth. Otherwise, we use sparse representations of the equations to show the quasimoothness of the family by using the computer algebra system MAGMA.

2.4 Computing invariants

If $R = \mathbb{C}[x_0, \ldots, x_N]/(f_1, \ldots, f_c)$ is the graded coordinate ring of a WCI X, with $\deg x_i = a_i$, then by [9, 3.4] the Hilbert series with respect to $D = \mathcal{O}_X(1)$ is the rational function

$$P_X(t) = \frac{\prod_{j=1}^{c}(1-t^{d_j})}{\prod_{i=0}^{N}(1-t^{a_i})} = \sum_{m\geq 0} h^0\left(X, \mathcal{O}_X(mD)\right) t^m. \tag{2}$$

Then the two invariants in Table 4 are computed as follows.

(i) *First plurigenus* $h^0(-K_X)$: The coefficients $h^0(X, \mathcal{O}_X(-mK_X))$ are called plurigenera of X and they are the invariants under smooth projective deformations [35]. We compute the first plurigenus $h^0(-K_X)$ as the coefficient of t^{i_X} in the Hilbert series expansion (2) where i_X is the Fano index of X.

(ii) *Intersection number* $(-K_X)^n$: It can be defined as an anticanonical degree of X which we calculate from the Hilbert series $P_X(t)$ of X. For an n-fold X we have

$$P_X(t) = \frac{H(t)}{(1-t)^{n+1}},$$

where $H(t)$ is a rational function with denominator and numerator having positive coefficients. Then for $D = \mathcal{O}_X(1)$ in the class group, we have $D^n = H(1)$. Consequently for a Fano manifold of index i_X, we have $(-K_X)^n = (i_X)^n D^n$.

3 Further analysis of results

In this section, we provide a summary of our analysis and proof. In particular, we give the details of the number of candidates in each dimension and how many of them are quasismooth. Also we provide a table that shows the distribution of Fano manifolds across various codimension, and another table which shows the distribution for each Fano index and corresponding dimension. We also provide a typical reason for the failure of a candidate Fano WCI to be smooth.

For each $n \in \{6, 7, 8, 9, 10\}$ we report the total number of weighted complete intersection *candidates* produced by the bounded search and, among them, the number that pass the smoothness checks (hence yield smooth Fano WCIs). In total, we obtain 1180 families of smooth Fano WCIs out of 1329 candidates Fano WCIs. As a sample we provide the list for dimension 6 in the Appendix 1 and provide all the tables on the GitHub page https://github.com/QureshiMI/Fano-WCI. Moreover, the degrees and weights for give wellformed WCI are unique.

n	6	7	8	9	10
Candidates	77	117	223	329	583
Smooth	72	108	201	292	507

We observe that, as the dimension increases, the ratio of smooth to candidate examples decreases. The ratios are 93.5%, 92.3%, 90.1%, 88.8% and 87%, in dimension 6, 7, 8, 9 and 10 respectively.

3.1 Distribution by codimension

In Table 2 we list, for each fixed dimension n, the distribution across codimensions c. For each dimension we have two rows, one showing the number of candidates #can and another #exps showing the number of examples of smooth varieties that actually exist. A zero shows no examples possible, as we know that if X is a Fano WCI then $c \leq n$ by [5, Theorem 1.6].

3.2 Distribution by Fano index

To complement the codimension view, we also provide a table whose *rows* are the Fano indices and whose *columns* are the dimensions n. This index–dimension perspective isolates how the supply of examples shifts with i_X across n, and it is particularly useful when comparing families at a fixed index. See Table 3.

3.3 Failed candidates

As we can see in Tables 2 and 3, not all candidates obtained by using the Hilbert series algorithm can be realized as smooth Fano WCI. Across the five dimensions $n = 6, \ldots, 10$, we find **149** candidates that fail: 5 in dimension 6, 9 in 7, 22 in 8, 37 in 9, and 76 in 10; see the Table 5 below for failures in dimension 6. We provide a complete list of failed candidates on https://github.com/QureshiMI/Fano-WCI.

Table 2 Candidates and smooth Fano WCIs by dimension n and codimension c.

Dim	c										
	1	2	3	4	5	6	7	8	9	10	Total
6 (#can)	18	33	17	6	2	1	0	0	0	0	77
6 (#exps)	18	31	15	5	2	1	0	0	0	0	72
7 (#can)	20	46	31	11	6	2	1	0	0	0	117
7 (#exps)	20	43	28	9	5	2	1	0	0	0	108
8 (#can)	27	78	66	32	11	6	2	1	0	0	223
8 (#exps)	27	73	58	26	9	5	2	1	0	0	201
9 (#can)	30	100	100	55	24	11	6	2	1	0	329
9 (#exps)	30	94	88	45	18	9	5	2	1	0	292
10 (#can)	35	152	184	114	54	24	11	6	2	1	583
10 (#exps)	35	145	158	93	41	18	9	5	2	1	507
Total (#can)	130	409	398	218	97	44	20	9	3	1	1329
Total (#exps)	130	386	347	178	75	35	17	8	3	1	1180

Table 3 Candidates and smooth Fano WCIs by dimension n and the index i_X of X

Di_X	1	2	3	4	5	6	7	8	9	10	Total
6 (#can)	40	14	14	4	4	1	0	0	0	0	77
6 (#exps)	37	13	13	4	4	1	0	0	0	0	72
7 (#can)	40	40	14	14	4	4	1	0	0	0	117
7 (#exps)	36	37	13	13	4	4	1	0	0	0	108
8 (#can)	106	40	40	14	14	4	4	1	0	0	223
8 (#exps)	93	36	37	13	13	4	4	1	0	0	201
9 (#can)	106	106	40	40	14	14	4	4	1	0	329
9 (#exps)	91	93	36	37	13	13	4	4	1	0	292
10 (#can)	254	106	106	40	40	14	14	4	4	1	583
10 (#exps)	215	90	94	36	37	13	13	4	4	1	507
Total (#can)	546	306	214	112	76	37	23	9	5	1	1329
Total (#exps)	472	269	193	103	71	35	22	9	5	1	1180

All these examples fail due to the contrapositive of the following Lemma.

Lemma 3.1 [14, Lemma 18.14] *Let* $X = X_{d_1,\ldots,d_k} \subset \mathbb{P}(a_0,\ldots,a_N)$ *be a quasismooth weighted complete intersection that is not a linear cone. Assume the degrees and the weights are ordered nondecreasingly:*

$$d_1 \leq \cdots \leq d_k \quad \textit{and} \quad a_0 \leq \cdots \leq a_N.$$

Then:

(i) For each $j = 1,\ldots,k$ *one has*

$$d_j > a_{N-k+j}.$$

Equivalently,

$$d_k > a_N, \quad d_{k-1} > a_{N-1}, \quad \ldots, \quad d_1 > a_{N-k+1}.$$

(ii) If $k \geq 2$ and $d_{k-1} < a_N$, then the top weight divides the top degree:

$$a_N \mid d_k.$$

Therefore if either of the above statement does not hold, then X is not quasismooth.

Corollary 3.2 *In the above setup, X is not quasismooth if either of the following two conditions is satisfied.*

(i) There exists $j \in \{1, \ldots, k\}$ with

$$d_j < a_{N-k+j}.$$

(ii) $k \geq 2$, $d_{k-1} < a_N$, and $a_N \nmid d_k$.

Note that the equality in the converse can not happen as X is not an intersection with a linear cone. One can use simple minded computer code to exclude such cases.

Appendix

See Tables 4 and 5.

Table 4 Smooth Fano 6-folds WCIs

S.No	c	i_X	Eq. degrees & Embedding	$(-K_X)^6$	$h^0(-K_X)$
1	1	1	$X_7 \subset \mathbb{P}(1^8)$	7	8
2	1	1	$X_8 \subset \mathbb{P}(1^7, 2)$	4	7
3	1	1	$X_9 \subset \mathbb{P}(1^7, 3)$	3	7
4	1	1	$X_{12} \subset \mathbb{P}(1^6, 3, 4)$	1	6
5	1	1	$X_{12} \subset \mathbb{P}(1^7, 6)$	2	7
6	1	1	$X_{14} \subset \mathbb{P}(1^6, 2, 7)$	1	6
7	1	2	$X_6 \subset \mathbb{P}(1^8)$	384	36
8	1	2	$X_{10} \subset \mathbb{P}(1^7, 5)$	128	28
9	1	3	$X_5 \subset \mathbb{P}(1^8)$	3645	120
10	1	3	$X_6 \subset \mathbb{P}(1^7, 2)$	2187	91
11	1	3	$X_8 \subset \mathbb{P}(1^7, 4)$	1458	84
12	1	3	$X_{10} \subset \mathbb{P}(1^6, 2, 5)$	729	62
13	1	4	$X_4 \subset \mathbb{P}(1^8)$	16,384	329
14	1	4	$X_6 \subset \mathbb{P}(1^7, 3)$	8192	217
15	1	5	$X_3 \subset \mathbb{P}(1^8)$	46,875	756
16	1	5	$X_4 \subset \mathbb{P}(1^7, 2)$	31,250	546
17	1	5	$X_6 \subset \mathbb{P}(1^6, 2, 3)$	15,625	336
18	1	6	$X_2 \subset \mathbb{P}(1^8)$	93,312	1386
19	2	1	$X_{2,6} \subset \mathbb{P}(1^9)$	12	9
20	2	1	$X_{2,10} \subset \mathbb{P}(1^8, 5)$	4	8
21	2	1	$X_{3,5} \subset \mathbb{P}(1^9)$	15	9

Table 4 continued

S.No	c	i_X	Eq. degrees & Embedding	$(-K_X)^6$	$h^0(-K_X)$
22	2	1	$X_{3,6} \subset \mathbb{P}(1^8, 2)$	9	8
23	2	1	$X_{3,8} \subset \mathbb{P}(1^8, 4)$	6	8
24	2	1	$X_{3,10} \subset \mathbb{P}(1^7, 2, 5)$	3	7
25	2	1	$X_{4^2} \subset \mathbb{P}(1^9)$	16	9
26	2	1	$X_{4,5} \subset \mathbb{P}(1^8, 2)$	10	8
27	2	1	$X_{4,6} \subset \mathbb{P}(1^7, 2^2)$	6	7
28	2	1	$X_{4,6} \subset \mathbb{P}(1^8, 3)$	8	8
29	2	1	$X_{4,10} \subset \mathbb{P}(1^6, 2^2, 5)$	2	6
30	2	1	$X_{5,6} \subset \mathbb{P}(1^7, 2, 3)$	5	7
31	2	1	$X_{6^2} \subset \mathbb{P}(1^6, 2^2, 3)$	3	6
32	2	1	$X_{6^2} \subset \mathbb{P}(1^7, 3^2)$	4	7
33	2	1	$X_{6,8} \subset \mathbb{P}(1^6, 2, 3, 4)$	2	6
34	2	1	$X_{6,10} \subset \mathbb{P}(1^5, 2^2, 3, 5)$	1	5
35	2	2	$X_{2,5} \subset \mathbb{P}(1^9)$	640	44
36	2	2	$X_{2,8} \subset \mathbb{P}(1^8, 4)$	256	35
37	2	2	$X_{3,4} \subset \mathbb{P}(1^9)$	768	45
38	2	2	$X_{4^2} \subset \mathbb{P}(1^8, 2)$	512	37
39	2	2	$X_{4,6} \subset \mathbb{P}(1^7, 2, 3)$	256	29
40	2	2	$X_{6^2} \subset \mathbb{P}(1^6, 2, 3^2)$	128	22
41	2	3	$X_{2,4} \subset \mathbb{P}(1^9)$	5832	156
42	2	3	$X_{2,6} \subset \mathbb{P}(1^8, 3)$	2916	113
43	2	3	$X_{3^2} \subset \mathbb{P}(1^9)$	6561	163
44	2	3	$X_{3,4} \subset \mathbb{P}(1^8, 2)$	4374	127
45	2	3	$X_{4^2} \subset \mathbb{P}(1^7, 2^2)$	2916	98
46	2	3	$X_{4,6} \subset \mathbb{P}(1^6, 2^2, 3)$	1458	69
47	2	3	$X_{6^2} \subset \mathbb{P}(1^5, 2^2, 3^2)$	729	47
48	2	4	$X_{2,3} \subset \mathbb{P}(1^9)$	24,576	441
49	2	5	$X_{2^2} \subset \mathbb{P}(1^9)$	62,500	966
50	3	1	$X_{2^2,5} \subset \mathbb{P}(1^{10})$	20	10
51	3	1	$X_{2^2,8} \subset \mathbb{P}(1^9, 4)$	8	9
52	3	1	$X_{2,3,4} \subset \mathbb{P}(1^{10})$	24	10
53	3	1	$X_{3^3} \subset \mathbb{P}(1^{10})$	27	10
54	3	1	$X_{3^2,4} \subset \mathbb{P}(1^9, 2)$	18	9
55	3	1	$X_{3,4^2} \subset \mathbb{P}(1^8, 2^2)$	12	8
56	3	1	$X_{4^3} \subset \mathbb{P}(1^7, 2^3)$	8	7
57	3	1	$X_{4^2,6} \subset \mathbb{P}(1^6, 2^3, 3)$	4	6
58	3	1	$X_{4,6^2} \subset \mathbb{P}(1^5, 2^3, 3^2)$	2	5

Table 4 continued

S.No	c	i_X	Eq. degrees & Embedding	$(-K_X)^6$	$h^0(-K_X)$
59	3	1	$X_{6^3} \subset \mathbb{P}(1^4, 2^3, 3^3)$	1	4
60	3	2	$X_{2^2,4} \subset \mathbb{P}(1^{10})$	1024	53
61	3	2	$X_{2^2,6} \subset \mathbb{P}(1^9, 3)$	512	43
62	3	2	$X_{2,3^2} \subset \mathbb{P}(1^{10})$	1152	54
63	3	3	$X_{2^2,3} \subset \mathbb{P}(1^{10})$	8748	199
64	3	4	$X_{2^3} \subset \mathbb{P}(1^{10})$	32,768	553
65	4	1	$X_{2^3,4} \subset \mathbb{P}(1^{11})$	32	11
66	4	1	$X_{2^3,6} \subset \mathbb{P}(1^{10}, 3)$	16	10
67	4	1	$X_{2^2,3^2} \subset \mathbb{P}(1^{11})$	36	11
68	4	2	$X_{2^3,3} \subset \mathbb{P}(1^{11})$	1536	63
69	4	3	$X_{2^4} \subset \mathbb{P}(1^{11})$	11,664	242
70	5	1	$X_{2^4,3} \subset \mathbb{P}(1^{12})$	48	12
71	5	2	$X_{2^5} \subset \mathbb{P}(1^{12})$	2048	73
72	6	1	$X_{2^6} \subset \mathbb{P}(1^{13})$	64	13

Table 5 Failed candidate Fano 6-fold WCIs

#	c	i_X	Variety
1	2	1	$X_{18,3} \subset \mathbb{P}(1^7, 6, 9)$
2	2	3	$X_{12,2} \subset \mathbb{P}(1^7, 4, 6)$
3	3	1	$X_{12,3,2} \subset \mathbb{P}(1^8, 4, 6)$
4	3	2	$X_{12,2,2} \subset \mathbb{P}(1^8, 4, 6)$
5	4	1	$X_{12,2,2,2} \subset \mathbb{P}(1^9, 4, 6)$

Author Contributions I did everything, being a sole author.

Data Availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare no conflict of interest.

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